# Mastering Dyalog APL <br> A Complete Introduction to Dyalog APL 

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# Introduction - Will You Play APL With Me? 

## Will You Follow Us?

We would like to have you discover a new land, a land where people who may or may not be specialists in programming can process their data, build computerised applications, and take pleasure in using a programming language which is an extremely elegant and powerful tool of thought.

## Beware: Dyalog APL is Addictive!

Among the hundreds of programming languages which have been created, most of them share the same fundamentals, the same basic instruction set, approximately the same functions, and by and large the same methods to control the logic of a program. This greatly influences the way people imagine and build solutions to computing problems. Because the languages are so similar, the solutions are similar. Does it mean that these are the only ways of solving problems? Of course not!

Dyalog APL is there to open doors, windows, and minds, prove that original new methods do exist, and that mathematics is not limited to four basic operations. Using APL will expand and extend the range of mental models that you use to solve problems, but beware:
Once you are hooked on APL, there is a real risk that you will no longer accept the limitations of "traditional" programming languages.

## Installation and Keyboard

If you do not have access to a computer with Dyalog APL installed, you should still be able to gain an appreciation of the language from these pages and, we hope, enjoy the experience.

If you have installed Dyalog, not only can you read this book, but you can also experiment on your own computer using the examples below, and invent your own data and calculations.
If you have a copy of Dyalog APL, install it as explained in the User Guide. Just run the installation program and accept all the defaults; there is no need to change anything.

You might like to refer to section A-1 for additional installation hints.
As you will see in the following pages, APL uses special symbols, like $\phi, \rho$, and $\downarrow$, which you enter using a special keyboard layout.
You will not need all of the special symbols to read the following pages. The picture below illustrates a cut-down version of the UK keyboard layout, with only the APL symbols that are referred to in this introduction. They are shown on a grey background. The US keyboard is slightly different, but the symbols we need are positioned identically. The full keyboard layout is shown in the User Guide.


Here is how the keyboard is to be used:

- All the standard English letters, numerals and symbols are typed as usual.
- The majority of the APL symbols are obtained by pressing the "Ctrl" key, in conjunction with another key. For example, to obtain $\rho$, you must press $C t r l$ and R. From now on, this keystroke will be identified as "Ctrl+R". On the keyboard layout illustrated above, the symbols that you enter this way are printed at the bottom-right corner of each key.
- Most other APL symbols are obtained by simultaneously pressing "Ctrl" and "Shift" and then the appropriate key. For example, to obtain $\Phi$, you must press Ctrl and Shift and 6. From now on, this keystroke will be identified as "Ctrl + Shift +6 ". On the keyboard layout illustrated above, the symbols that you enter this way are printed at the top-right corner of each key.
- A few APL symbols are obtained by pressing the "Alt Gr" key (or Ctrl+Alt if your keyboard does not have an "Alt Gr" key), simultaneously with another key, but you don't have to bother about them here: We won't be using them in the examples in this introduction.
- In case you initially have any difficulty with the keyboard, there is a "language bar" on top of the session screen, with all the APL symbols on it. When you need a symbol, you just have to click on it and the symbol will appear wherever your cursor is positioned.

You may notice that some symbols appear twice on the keyboard. This is the case for example for the symbols < = >. These symbols are all part of a normal keyboard (the black ones), but they have been repeated on the APL keyboard, mostly in order to group the symbols used for comparison functions together (the red ones). Do not worry: No matter which key you use to produce one of the duplicated symbols, you'll obtain the same result.

## Utilities and Data

For most of the examples, you can just type what you read in the following pages, but sometimes you will need some data which we have prepared for you.

This data is contained in a special file (called a WorkSpace) named DyalogTutor_EN.dws which accompanies this book.
If you don't have the file, please refer to section A-1 for instructions on how to download it.
The file can only be opened by Dyalog APL. You can open it as follows:

- In Windows Explorer, double-click on the file's name. APL will be started, and it will then open the file.
- Or, start APL by double clicking on the Dyalog APL icon. Then, using the "File/Open" menu, search for the file and open it.

Once the workspace has been loaded, a welcome message is displayed, and you can check that the workspace contains the variables we shall be using in the following pages:

```
    )vars
Actual Ages AlphLower AlphUpper Area Big Category etc...
```

You can display the contents of any variable by hovering over its name with the mousepointer, by double-clicking on its name, or just by typing its name and pressing the Enter key, like this:

```
    Forecast
```



```
300}3330 360 400 500 52
100}25
    50}120\quad220 300 320 35
```

Now, you are ready, fasten your seatbelts, we're off!

## Our First Steps into APL's Magic World

## Simple Operations

In APL, what you type starts 6 characters right from the left margin (we say it is "indented"), whereas the computer's response begins at the left margin. For additional clarity, in the following pages the characters typed by the user are printed in red, the response given by the computer being in black.

You will notice that in the examples given in the book we very often put a blank space between a symbol and the surrounding names or values. This is in most cases unnecessary; we only do so in order to improve readability. Later on, we will gradually cease to insert the blank spaces in expressions that you should become familiar with along the way.
An expression gets evaluated and the result shown in the session when you press the Enter key. Let's try some simple expressions:
$27+53$
80
1271-708
563
$86 \div 4$ The Divide sign is obtained using Ctrl+ $=$
21.5
$59 \times 8$ The Multiply sign is obtained using Ctrl+ -
472
You can see that APL behaves like any hand-held calculator with, however, a small difference; multiplication is represented by the multiplication symbol ( $\times$ ) which is used in schools in many countries; likewise for division ( $\div$ ).

In most other computer languages, a star * is used for Multiply and / for Divide. This is a legacy of the early days of computers, when the character set was limited to the one available on a typewriter. At the time it was decided to use $*$ and $/$ in place of $\times$ and $\div$. But it is now possible to display any type of symbol on a screen and on a printer, and this transposition is no longer justifyable. The use of common symbols, which are taught all over the world, aids the understanding of APL by non programmers.

If you are familiar with other programming languages, you may occasionally and erroneously use $*$ instead of $\times$. Let's see what might happen then:
$7 \times 3$
343
$\checkmark$ In APL the star means "Power" so that $7 * 3$ is equivalent to $7 \times 7 \times 7$

## Variables

As in any programming language, it is possible to create variables. Just choose a name and use the left arrow to assign it a value. In APL a numeric value can consist of a single number, or several numbers separated by at least one blank space. The arrow can be obtained using Ctrl+ [

```
VAT * 19.6
\diamond ~ R e a d ~ i t ~ a s : ~ V A T ~ g e t s ~ 1 9 . 6 ~
Years \leftarrow19521943 1986 2007
```

The names are "case sensitive". It means that three variables named respectively VAT, Vat, and vat, would be distinct, and may contain different values.

To ask for the contents of a variable, just type its name and press Enter, like this:
VAT
19.6

Years
1952194319862007

## Array Processing

APL is able to operate on two sets of numbers, provided those two sets have the same "shape". For the moment, understand this as "the same number of items". For example, suppose that you have a list of prices of 5 products, and the quantity bought of each:

| Prices | 5.20 | 11.50 | 3.60 | 4.00 | 8.45 |
| :--- | :---: | :---: | :--- | :--- | :--- |
| Quantities | 2 | 1 | 3 | 6 | 2 |

You can create two variables like this:

```
Price + 5.2 11.5 3.6 4 8.45
Qty & 2 1 362
```

When multiplied together, the variables are multiplied item by item, and produce a result of the same length. That result can be assigned to a new variable.

Costs $\leftarrow$ Price $\times$ Qty
Costs
10.411 .510 .82416 .9

This array processing capability eliminates most of the "loops" which are common to other programming languages. This remains true even if the data is not a simple list but a multidimensional array, of almost any size and number of dimensions.

To make it clear, imagine that a Sales Director makes forecasts for sales of 4 products over the coming 6 months, and assigns them to the variable Forecast. At the end of the 6 months, he assigns the real values to the variable Actua 1. Here they are:

| $c$ | Forecast |  |  |  |  |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 150 | 200 | 100 | 80 | 80 | 80 | 141 | 188 | 111 | 87 | 82 | 74 |
| 300 | 330 | 360 | 400 | 500 | 520 | 321 | 306 | 352 | 403 | 497 | 507 |
| 100 | 250 | 350 | 380 | 400 | 450 | 118 | 283 | 397 | 424 | 411 | 409 |
| 50 | 120 | 220 | 300 | 320 | 350 | 43 | 91 | 187 | 306 | 318 | 363 |

We have not yet explained how you can build such arrays of data, but if you have APL installed, these variables are provided in the Workspace file named "DyalogTutor_EN.dws". Refer to the "Utilities and Data" section above to see how you can load the workspace and access the data.

It is clear that the first idea of any Sales Director will be to ask for the differences between what he expected and what he has really got. This can be done easily by typing:

|  | Actual - Forecast |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| -9 | -12 | 11 | 7 | 2 | -6 |
| 21 | -24 | -8 | 3 | -3 | -13 |
| 18 | 33 | 47 | 44 | 11 | -41 |
| -7 | -29 | -33 | 6 | -2 | 13 |

$\diamond$ Note that to distinguish the sign attached to negative values from subtraction, negative values are shown with a high minus sign.

To enter negative values, this high minus sign can be obtained by pressing CtrI +2 .
In most traditional programming languages an operation like the one above requires two embedded loops. See what is needed in PASCAL:

```
DO UNTIL I=4
    DO UNTIL J=6
        DIFF(I,J):=ACTUAL(I,J)-FORECAST(I,J)
    END
END.
```

Even if this may seem obvious to a programmer, it is worth noting that most of the code has nothing to do with the user requirement. The only important thing (subtract forecasts from actual values) is hidden behind the detailed workings of the computer program.

To have a calculation done by a machine, one must translate our human wording into something that the computer can understand. With traditional languages, most of that effort is made by the man, to produce a program like the PASCAL example above. The great advantage of APL is that the man has generally much less effort to make, and the machine does the rest.

We have seen that APL will work on two variables of the same shape; it also works if one of the variables is a single item, which is called a scalar. If so, the other variable may be of any shape.

For example, if we want to calculate the amount of $19.6 \%$ VAT applied to the variable Price above, we can type Price $\times$ VAT $\div 100$ (or VAT $\times$ Price $\div 100$ as well), as shown here:

```
    Price x VAT \div 100
1.0192 2.254 0.7056 0.784 1.6562 ↔ This result would require some rounding
                                    but this is not important for now
```


## More Symbols

Most programming languages represent only a very small subset of the mathematical functions using symbols (typically,+- , and $/$ ). The creator of APL, Kenneth E. Iverson, chose to include many traditional mathematical symbols in his language, and also added some new symbols to the set that we already know so well.
E.g.: Many functions which in other programming languages are library routines with names like "Maximum" have their own symbols in APL.
The function "Maximum" ( $\Gamma$ ) returns the greater of two numbers, or of two arrays of numbers compared item by item.

There is also, as one might expect, a symbol for "Minimum" ( L ).

```
    75.6 「 87.3 Maximum \((\mathrm{Ctrl}+\mathrm{S})\)
```

87.3
$\begin{array}{llllllll}11 & 28 & 52 & 14 & \text { 「 } 30105020 & 6 & \\ \text { Comparison item by item }\end{array}$
30285220
$\begin{array}{llll}11 & 28 & 52 & 14 \text { L } 20 \text { Minimum ( } \mathrm{Ctrl}+\mathrm{D} \text { ) })\end{array}$
11202014
APL supports about 70 symbols. Since some symbols have more than one meaning one could argue at length about the exact number.
This is nothing to worry about: Some of the symbols are familiar; such as $\times$ or $>$ or again $\div$ and -, but also! and a good many others.

## Most Symbols Have a Double Meaning

This is not a peculiarity of APL; in algebra we are familiar with the use of symbols as common as the minus sign being used in two different ways.
$\begin{array}{lll}\text { In the expression } & a=x-y & \text { the minus sign means subtract } \\ \text { Whereas in } & a=-y\end{array} \quad$ the minus sign indicates the negation of $y$, that's different
The first form is called the "dyadic" use of the symbol.
The second form is called the "monadic" use of the symbol.
It is the same in APL, where most of the symbols can have two meanings.
For example, to find the shape (the dimensions) of an array, one uses the Greek letter Rho ( $\rho$ ), which can be read "shape of ...", in its monadic use. It is produced using Ctrl+R.

```
    \rho Price & Monadic use
5
    \rho Forecast
46
    Forecast has 4 rows of 6 items
```

Used dyadically, the same symbol will organise items into a specified shape. For example, suppose that we want to create the matrix below:

$$
\begin{array}{ll}
25 & 60 \\
33 & 47 \\
11 & 44 \\
53 & 28
\end{array}
$$

We must give the computer two pieces of information:

- First the shape to give to the matrix: 42 (4 rows of 2 columns)
- Next the contents of the matrix: $\quad \begin{array}{llllllll}25 & 60 & 33 & 47 & 11 & 44 & 53 & 28\end{array}$

It is the symbol $\rho$ (Rho) which makes the connection between the shape and the contents:

```
    Tab +4 2 \rho 25 60 33 47 11 44 53 28
```

    Tab
    2560
3347
1144
5328

A new variable Tab is thereby created, and this is also how the variables Forecast and Actual above were made.

## Conventions

In APL, we give special names to certain shapes of data:

- Scalar is used for a single value, a number like 456.18 or a single letter like 'Q'.
- Vector is a plain list of values It may be composed of numbers like Price and Qty, or of letters like 'Once upon a time' within single quotes
- Matrix is an array with two dimensions, like Forecast or Tab
- Array is a generic word for any set of values, whatever the number of its dimensions
- Table is a common word used for arrays with 2 dimensions (matrices)
- Cube is a common word used for arrays with 3 dimensions


## Reduction Unifies Traditional Notations

```
Perhaps you remember the variable Costs: }\quad\begin{array}{llllllll}{10.4}&{11.5}&{10.8}&{24}&{16.9}
```

So what must we do to work out the total? Mathematicians are creative people who long ago devised the symbol $\sum$ always with a pretty collection of indices above and below, which make it complex to understand and to type on a typewriter.

In APL, the operation is written like this:

```
    +/ Costs
```

73.6

Simple isn't it? This gives the total of all the items of the array.
You can read this as "Plus Reduction" of the variable Costs.
To gain a better understanding of the process:
When we write an instruction such as
+/ $214518 \quad 2711$

- it works as if we had written
$21+45+18+27+11$
- and we obtain the sum 122

In fact, it works as if we had "inserted" the symbol + between the values.

But then, if we write

- it is as if we had written
- so, we get the product
$\times / 2145182711$
$21 \times 45 \times 18 \times 27 \times 11$
5051970
Similarly，if we write
「／ $214518 \quad 2711$
－it is as if we had written
21「 45「18「27「11
－so，we obtain the largest term
45

Reduction，represented by the symbol／，belongs to a special category of symbols called Operators．All the other symbols（ + －「 $\rho 申 \ldots$ ．．）are called Functions（addition， subtraction，multiplication，maximum，shape，etc．）．
The arguments of a function are data（arrays）：Price $\times$ Qty
Whereas at least one of the arguments of an operator is a function：＋／Qty
The left argument of Reduction can be one of many of the APL symbols，and it can also be the name of a user－defined program．This may give you an idea of the generality and power of the concept．
Dyalog APL contains 10 such powerful operators．If that is not enough，you can even write your own operators，just like you can write your own functions！

## Let＇s Write Our First Programs

Imagine that we want to calculate the average of the following numbers：

```
Val * 22 37 41 19 54 11 34
```

We must：
－first calculate the sum of the values：＋／Val giving 218
－next calculate the number of values：$\rho$ Val giving 7
－and finally divide one result by the other
The calculation can be written as the single formula：$\quad(+/ \vee a 1) \div(\rho \vee a 1)$
As it is quite likely that we shall often want to make this sort of calculation，it is preferable to store this expression in the form of a program．

In APL we prefer the name defined function to the name＂program＂．
Defined functions may be used in the same way as the built－in functions represented by special symbols like $+-\times->\rho \ldots$ ，which are called primitive functions．
To define a simple function like this one，here is the easiest way：

$$
\text { Average } \leftarrow\{(+/ \omega) \div(\rho \omega)\}
$$

## Average

$\omega$
$\alpha$
is the program name
is a generic symbol which represents the array passed on the right. would be the generic symbol for the array passed on the left, if any

The definition of the function is delimited by a set of curly braces \{ and \}. For more complex functions it is also possible to use a text editor, but this is beyond the scope of this short introduction.

Once defined, this function may be invoked in a very simple way:

Average Val $\langle$ For execution, $\omega$ will get the values
31.1428571428

Average 12745623
41.25

Let us also write two little dyadic functions, the left argument of which is $\alpha$, and the right is $\omega$ :
Plus $\leftarrow\{\alpha+\omega\}$
Times $\leftarrow\{\alpha \times \omega\}$
(3 Plus 6) Times (5 Plus 2)
63
As you can see, these functions behave exactly as if we had written $(3+6) \times(5+2)$
We said in the preceding section that a user-defined program could be used by the Reduce operator; let us try:

Plus/ Val
218
« It works!

## Indexing

$$
\left.\begin{array}{lllllllll}
\text { Returning to our vector of numbers Val: } & 22 & 37 & 41 & 19 & 54 & 11 & 34
\end{array}\right] \begin{array}{ll}
\text { In order to extract the 4th item, we just write: } & \text { Val [4] }
\end{array}
$$

In many other programming languages one uses parentheses instead of brackets; this is not very different.

What is new is that one can extract several items in one instruction.

```
    Val
22 37 41 19 54 11 34
    Val[[2 4 4 7 1 4 4 ] O
37 19 34 22 19
```

And of course, in the same way, one may modify one or more items of val using their indexes. Naturally, one must provide as many values as there are items to modify, or a single value for all:

```
    Val[3 5 1] + 0
    Val
```

$\begin{array}{lllllll}0 & 37 & 019 & 0 & 11 & 34\end{array}$
Val[3 51$]+30077111$
Val


It is often necessary to extract the first few items from a list of values, for example the first 5 . Nothing could be easier:

```
    Val[1 2 3 4 5]
111 37 300 1977
```

But if one needs to extract the first 500 items from a long list, typing the integers from 1 to 500 would of course be very inconvenient.

This is why APL has been given the symbol $\imath$ (Iota), which produces the set of the first $n$ integers ( 2 can be obtained using $C t r l+\mathrm{I}$ )
Thus, instead of writing $12 \begin{array}{llllll} & 4 & 6 & 7 & \text {, it is sufficient to write } 28 \text {. }\end{array}$
And to extract the first 500 terms of a large vector, one may write: Big [ 1500 ]
We shall discover later an even simpler method.

## Calculating Without Writing Programs

The employees of a company are divided into three hierarchical categories, denoted simply 1 , 2 , and 3 . One assigns to two variables the salaries and the categories of these employees; as partly shown here:


Do they never want to increase these salaries? (what has our poor world come to!).

A rumour reaches us about their plans: They want a different percentage increase for each category, according to the following scale:

| Category | Suggested <br> increase |
| :---: | :---: |
| 1 | $8 \%$ |
| 2 | $5 \%$ |
| 3 | $2 \%$ |

How much is this going to cost the company?
We create a variable containing the above three rates:

```
    Rates * 8 5 2 % 100 & APL allows us to divide three numbers by a single one
    Rates
0.08 0.05 0.02
```

The first employee is in category 3, so the rate that applies to him is:
Rates[3]
0.02

It follows that the first 5 employees, being in categories 31322 respectively, are entitled to the following increases:

```
    Rates[3 1 3 2 2]
0.02 0.08 0.02 0.05 0.05
```

More generally, the rates applied to all of our employees could be obtained like this:
Rates[Categories]
$0.020 .08 \quad 0.020 .050 .050 .08 \quad 0.020 .020 .020 .05 \quad 0.05 \quad 0.02$ etc.
Having the rates, one has just to multiply by the salaries to obtain the individual increases:

```
    Salaries }\times\mathrm{ Rates[Categories]
84.5 129.52 74.12 112 103.8 1111.12 78.32 78.36 98.78 136.75 etc.
```

Finally, by adding them all, one will know how much it will cost the company:

```
    +/ Salaries \times Rates[Categories]
```

2177.41

You may note that:

- The expression remains valid whatever the number of employees or categories,
- the result has been obtained without writing any program,
- and this expression can be read as the simplest possible English, like this:

Sum the Salaries multiplied by Rates according to Categories
Clever, no?
This illustrates how the expression of a solution in APL can be very close to the way that the solution could be phrased in everyday language. This also shows clearly that the ways of reasoning induced by traditional programming languages are not the only possible ones. This difference and originality, introduced by APL, are among the major features of the language.

## Friendly Binary Data

APL makes much use of binary data. It is most often created by means of relational functions like $=$ or $>$, which give the answer 1 or 0 , depending whether the relation is true or not:

Salaries > 3000

Actual > Forecast
001110
$100100 \quad \leftrightarrow$ One can see the favourable results instantly
$\begin{array}{llllll}1 & 1 & 1 & 1 & 1 & 0\end{array}$ 000101

APL offers the conventional mathematical form of the 6 relational functions:


Naturally one can operate on this binary data using all the functions of Boolean algebra, and moreover, the symbols used are those familiar to mathematicians of all nationalities around the world:

Function $A N D$ is represented by the symbol $\wedge \quad$ (represented by the word AND in many programming languages)
Function $O R \quad$ is represented by the symbol $\quad v \quad$ (represented by the word $O R$ in these languages)

Thus, if I am looking for people in category 3 whose salary is less than 4000 euros, I can write:
(Categories $=3$ ) ^(Salaries < 4000)
00100011000000010001
In fact APL offers all the functions of Boolean algebra, including some perhaps less familiar functions like NOR and NAND (Not-OR and Not-AND), but they are very useful in finance and electronic automation.

There is, however, no special symbol for the function Exclusive OR (often called XOR). This is because it is not needed: The function Not Equal $\neq$ gives the same result as Exclusive OR when it is used with Boolean values, as you can see below:

```
    0 0 1 1 # 0 1 0 1
0110
```

Finally, not only can these binary vectors be used as we have described but also for novel purposes, such as counting and selecting.

## Counting

Having found which salaries are less than 2500 euros by means of the following expression:
Salaries < 2500
01011110000000110001010
It is easy to add all the 1 s and 0 s to calculate how many people earn less than 2500 euros:
+/ Salaries < 2500
8

## Selection

One can also use the binary vector as a "mask" to select the items corresponding to the binary " 1 "s from another array:

```
    1 1 0 1 0 0 1 / 23 55 17 46 81 82 83
23 55 46 83
```

The procedure is identical for character data:

```
    1010000 1 1 / 'Drumstick'
```

Duck
This function, called Compress, is particularly useful for extracting the items conforming to a given criterion from a variable. For example, to display the salaries of people in Category 2, one writes:
(Categories $=2$ ) / Salaries
$224020762735327813393319 \quad$ ¿ Powerful, isn’tit?

## Discovery

To practise our skills some more, let us find in our variable Val the positions of numbers greater than 35 . Here are the necessary steps:


Let us compare two of these results

| $\mathrm{Val}>35$ | $\Rightarrow$ | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| ipVal | $\Rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

You can see that that if you eliminate (using Compress) the items which correspond to zeros in order to retain only those corresponding to 1 , you easily get the positions required: 235

Thus the job may be done as follows:

```
    (Val>35) / \imath\rhoVal
235
```

This expression is applicable in many different situations.
Here is a similar use, but applied to character data: To find the positions of "a" within a phrase; the method is the same.

```
Phrase & 'Panama is a canal between Atlantic and Pacific'
(Phrase = 'a') / \imath\rhoPhrase
2 4 6
```


## A Touch of Modern Math

Proudly having found all the "a"s, we may wish to find all the vowels.
Alas, although we can write Phrase $=$ ' a ', because a vector can be compared with a single value, one cannot write Phrase $=$ 'ae iouy ${ }^{\prime(1)}$, because that would require the item by item comparison of a phrase of 46 letters and "aeiouy" which has only 6 .
In other words: You may compare 46 letters with 46 other letters, or compare them with one letter, but not with 6 .

So we shall use a new function: Membership which is represented by the symbol $\epsilon$, also used in mathematics. ( $\epsilon$ can be obtained by pressing Ctrl+E)

The expression $A \in B$ returns a Boolean result which indicates which items of the variable $A$ appear in the variable B, wherever they may be. And it works no matter what are the shapes, the dimensions or the type (numeric or character) of $A$ and $B$, a pure marvel!

For example:


[^0]So in pursuit of our enquiry we shall write:

```
    (Phrase \epsilon 'aeiouy') / \imath\rhoPhrase
2 4 6 8 111 14 16 20 23 24 30 33 36 41 43 45
```

One can also use membership between a vector and a matrix, as shown below, assuming that the list of towns is a variable created earlier.

We have represented side by side the variable itself and the result of using Membership:

```
    Towns
Canberra
Paris
Washington
Moscow
Martigues
Mexico
```

|  | Towns |  |  |  |  |  |  | $\epsilon$ | 'aeiouy' |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |

We can reverse the expression, but the result has always the same shape as the left argument:

```
        'aeiouy' \(\epsilon\) Towns
111110
\(\hookleftarrow\) None of the town names contains a "y"
```


## A Powerful Search Function

We have harnessed a very useful method to look for the positions of letters or numbers in a vector, but the answer obtained does not provide a one to one correspondence between the search values and the resultant positions:

```
    List & 154063182740 33 29 40 88 ৫ Vector of values
    Where + 2963403350 ↔ We want to find these
    (List \in Where) / \imath\rhoList & Let's apply our method
```

$\hookleftarrow$ Vector of values
$\leftrightarrow$ We want to find these
$\hookleftarrow$ Let's apply our method
$\hookleftarrow$ Positions found
236789

The positions are correct, but 29 is not in position 2 , and 40 is not in position 6 .
The question we have answered using the expression above is: "In which positions in List do we find a number that also appears somewhere in Where?"

If we want to answer the slightly different question: "Where in List do we find each number in Where?" we need to use a different method.

This new method uses the dyadic form of the symbol 2 (Iota).

```
    List \leftarrow 15 40 63 18 27 40 33 29 40 88 ↔ Same vector of values
    Where \leftarrow2963403350 \diamond \ Where are these?
    List \imath Where \diamond New method using dyadic \imath
    \bulletPositions found
```

832711

It is true that 29, 63, 40 and 33 , occur respectively in positions 8,3,2 and 7. It's much better!
But, first surprise: The value 40 occurs 3 times in List, but only the first one is reported in the result. This is because, by definition, dyadic Iota returns only the first occurrence of a given item. If the response for each value sought has to match a position; how may one, looking for 5 numbers, obtain 7 results?
Second surprise: The value 50 is reported as being found in position 11 in a vector comprising only 10 items! This is how the function IndexOf (dyadic 乞) reports that a value is absent.
At first sight this seems a bit surprising, but in fact it is a property which makes this function so generally powerful, as we shall soon see.

## An Example

A car manufacturer decides that he will offer his customers a discount on the catalogue price (you can see how this example is imaginary!)
The country has been divided into 100 areas, and the discount rate will depend on the area according to the following table:

| Area | Discount |
| :---: | :---: |
| 17 | $9 \%$ |
| 50 | $8 \%$ |
| 59 | $6 \%$ |
| 84 | $5 \%$ |
| 89 | $4 \%$ |
| Others | $2 \%$ |

The problem is to calculate the discount rate that may be claimed for a potential customer who lives in given area $D$; for example $D \leftarrow 84$.

Let us begin by creating two variables:

Area | 17 | 50 | 59 | 84 | 89 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Discount | + | 8 | 6 | 5 | 4 | 2 |

Let us see if 84 is in the list of favoured areas:
D $\in$ Area
1
$\hookleftarrow$ Yes, it's there
Area ¿ D
4 - 84 is the 4 th item in the list
Null Item ..... 505
N -Wise Reduce ..... 380
O
Object Representation ..... 495, 511
Object Size ..... 487
OLE ..... 675
Operators ..... 105, 377
Or (function) ..... 100
Orange ..... 449
Order of Evaluation ..... 94
OrIf (flow control) ..... 170
Outer Product ..... 386
Overtaking ..... 301
$P$
Page Designer (NewLeaf) ..... 726
Page Width ..... 508
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[^0]:    ${ }^{1}$ " Y " is considered to be a vowel in many European languages.

